

# Temporal Multilayer Network Analysis for Marvel Superheroes

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1 **Network analysis is a systematic way of studying movie (1) or comic  
2 character relationships. Applying network techniques enables re-  
3 searchers to discover hidden community structures that can be sur-  
4 prising to fans. We construct a temporal multilayer network to study  
5 how relationships among Marvel superheroes change across differ-  
6 ent eras since the 1950s. Preliminary numerical experiments show  
7 initial success of our method, but a more sophisticated edge weight-  
8 ing scheme is needed.**

Temporal Network | Multilayer Network | Social Network Analysis |  
Comic Character Analysis

## 1 Introduction

2 In recent years, using network analysis to study community  
3 structures of movie (1) or comic characters has gained popularity.  
4 The main characters can be obvious sometimes, but it is harder to draw a conclusion for a franchise with multiple  
5 episodes and storylines (2). Therefore, applying network  
6 techniques such as centrality measures enables researchers  
7 to discover hidden community structures. Comic character  
8 analysis derives many of its techniques from movie character  
9 analysis, pioneered by computer scientists who led the early  
10 developments of facial recognition algorithms (1). One can  
11 think about comics as simplified versions of movies in our  
12 context because movie dialogues and visuals are harder to parse.  
13 The richer display of emotions also involve careful sentiment  
14 analysis to thoroughly study a movie character network. Both  
15 comic and movie character network are part of the broader  
16 study of Social Network Analysis (SNA), and they exhibit  
17 properties similar to real-world social networks like a Facebook  
18 friendship network.

19 In our paper, we study the community structure of Marvel  
20 superheroes over time, namely, the different periods of comic  
21 book development. We use a temporal multilayer network  
22 to divide heroes into different layers: each layer contains all  
23 heroes that appear in comics released in an era. Within a layer,  
24 each node represents each character, and an edge exists be-  
25 between two nodes if the two characters have interacted. Across  
26 different layers, we use the convention that an inter-layer edge  
27 joins the character itself if it appeared in different periods.  
28 Comic books have significantly different common themes across  
29 eras due to social changes. For example, the Bronze Age that  
30 lasted from 1970 to 1985 focused on reflecting issues like racism  
31 in its storylines (3). Temporal multilayer network is still a new  
32 approach in analyzing comic characters, and we want to see  
33 if it helps us to understand whether character relationships  
34 were affected by stylistic changes.

## 36 Relevant Work

37 Temporal multilayer network is a new approach of studying  
38 comic character relationships, and past works have been lim-

ited. Existing works revolve around movie character analysis,  
39 which is very similar to comic analysis, so we used it as the ba-  
40 sis of our exploration. They use a traditional, one-dimensional  
41 node-edge structure to construct networks, where a node rep-  
42 presents a character, and an edge exists if the two characters  
43 have interacted. Basic degree centralities (2) were used to  
44 assign communities, which is not always reliable for complex  
45 networks.

46 To collect character co-appearance data, these studies organize  
47 researchers and volunteers to watch the movies or read the  
48 comics, and take an “average” of their responses to decide if  
49 an interaction exists (2). In addition, they use sophisticated  
50 facial recognition or computer vision algorithms (1) to detect  
51 a change in scenes so that researchers can assign sentiment  
52 score of each interaction to weigh the edges. As we can see,  
53 this approach is very labor-intensive and is impossible for large  
54 networks. Although their results correctly predict character  
55 relationships most of the time (1), this approach is error-prone  
56 since it relies on human judgement to assign sentiment scores.  
57 Despite having a cross-validation step to normalize across all  
58 responses, the interpretation can still be biased if there are  
59 only a few researchers available.

## Significance Statement

60 Past papers on movie (1) or character network analysis have  
61 been one-dimensional, ignoring the time component when eval-  
62 uating how network structures change. They use degree cen-  
63 tralities as thresholds to cluster characters into major, minor, or  
64 extra roles(2). This is computationally efficient, but doesn't work  
65 well for large, complex networks. In addition, they rely on the  
66 assumption that the characters can usually be clustered into  
67 two or three communities each led by a leading character(1).  
68 This motivates the goal of our project: we want to consider  
69 temporal component of a network and use more sophisticated,  
70 eigenvector-based centrality methods and Louvain algorithms  
71 for community detection.

72 Allen did research on prior works in comic and movie network analysis and worked on community  
73 detection with Ruiyao. He compared and contrasted our result with past works. He wrote the Abstract,  
74 Introduction, Significance Statement, and Conclusions and Discussion. Amy pre-processed  
75 the data and worked on visualization of multilayer network. She wrote the the centrality and tem-  
76 poral network definitions and data pre-processing sections of the report. Hercy helped determine  
77 and layers of the temporal network, researched supracentrality analysis of temporal networks, and  
78 ran experiments on them. She also wrote the definitions of supracentrality matrix construction, joint  
79 and conditional centralities, and all the other supracentrality methods section on the final report and  
80 the presentation. Owen did research on visualization of temporal networks. He summarized the  
81 output of community detection, interpreted them, and wrote about real-life explanations of these  
82 results. Jim researched the available datasets, and cleaned and processed the original dataset.  
83 He also created visualizations for the general dataset and visualizations for the centrality mea-  
84 sures. He wrote the Data Visualization and the Temporal Network Centrality Visualization sections  
85 on the final report and the presentation. Ruiyao did research on multilayer community detection  
86 and worked on community detection with Allen. She also worked on the visualization of the parti-  
87 tion result. She wrote the background, methods, some parts of results, and limitations section of  
88 community detection.

## 61 **Background**

### 62 **A. Brief overview of centrality measures.**

63 **Degree Centrality.** The degree centrality of a node  $i$  is simply its  
64 degree, often denoted  $k_i$ :

$$65 \quad k_i = \sum_{j=1}^n A_{ij} \quad [1]$$

66 where  $A_{ij}$  is the  $ij$ th entry of the adjacency matrix of the  
67 network, and  $n$  is the number of nodes(4).

68 **Eigenvector Centrality.** The eigenvector centrality of a node  $i$   
69 is its corresponding entry in the leading eigenvector of the  
70 adjacency matrix:

$$71 \quad x_i = \kappa^{-1} \sum_{j=1}^n A_{ij} x_j \quad [2]$$

72 or in matrix form:

$$73 \quad \mathbf{Ax} = \kappa \mathbf{x} \quad [3]$$

74 where  $\mathbf{A}$  is the adjacency matrix of the network and  $\kappa$  is  
75 the corresponding eigenvalue for the leading eigenvector  $\mathbf{x}$ (4).  
76 This centrality measure rewards a node for the importance of  
77 the nodes that it connects to.

78 **PageRank Centrality.** The PageRank centrality of a node  $i$  is:

$$79 \quad x_i = \alpha \sum_{j=1}^n A_{ij} \frac{x_j}{k_j^{out}} + 1 \quad [4]$$

80 or in matrix form:

$$81 \quad \mathbf{x} = (\mathbf{I} - \alpha \mathbf{AD}^{-1})^{-1} \mathbf{1} \quad [5]$$

82 where  $\alpha$  is a predetermined parameter with  $\alpha < \frac{1}{\lambda_1(\mathbf{AD}^{-1})}$   
83 ( $\lambda_1(\mathbf{AD}^{-1})$  is the largest eigenvalue of  $\mathbf{AD}^{-1}$ ), and  $k_j^{out}$  is  
84 the out-degree of node  $j$ (4). This centrality measure rewards  
85 a node for the importance of the nodes that it connects to,  
86 but for an amount inversely proportional to their out-degrees,  
87 and it gives each node a "free" centrality value of 1.

88 **Closeness Centrality.** The closeness centrality is the inverse of  
89 the mean distance from a node to other nodes:

$$90 \quad C_i = \frac{1}{l_i} = \frac{n}{\sum_j d_{ij}} \quad [6]$$

91 where  $d_{ij}$  is the shortest distance from node  $i$  to node  $j$ (4).

92 **Betweenness Centrality.** The betweenness centrality measure the  
93 extent to which a node lies on the paths between other nodes,  
94 and it is defined to be:

$$95 \quad x_i = \sum_{st} \frac{n_{st}^i}{g_{st}} \quad [7]$$

96 where  $n_{st}^i$  is the number of shortest paths from node  $s$  to  
97 node  $t$  that pass through  $i$ ,  $g_{st}$  is the total number of shortest  
98 paths from node  $s$  to node  $t$ , and we let  $n_{st}^i/g_{st} = 0$  if both  
99  $n_{st}^i = g_{st} = 0$ (4).

**B. Temporal networks.** A temporal network is a special type  
100 of multilayer network (which is a set of individual networks,  
101 each with its own nodes and edges, plus some edges between  
102 different layers), in which each layer represents the network at  
103 a different point or interval of time(4).  
104

**C. Single-layer modularity maximization.** Consider an  $\mathbf{N}$ -node  
105 network  $\mathbf{G}$  and let the edge weights between pairs of nodes  
106 be  $\{\mathbf{A}_{ij} \mid i, j \in \{1, \dots, N\}\}$ , so that  $\mathbf{A}$  is the adjacency  
107 matrix of  $\mathbf{G}$ . Suppose that there is a partition  $\mathbf{C}$  of a network  
108 into  $\mathbf{K}$  disjoint sets of nodes  $\{\mathbf{C}_1, \dots, \mathbf{C}_K\}$ . We can define  
109  $\mathbf{c}(\mathbf{i}) = \mathbf{c}(\mathbf{j}) = \mathbf{k}$  if and only if  $\mathbf{i}$  and  $\mathbf{k}$  lie in the same  $\mathbf{C}_k$ . The  
110 value of modularity for a given partition  $\mathbf{C}$  is then  
111

$$112 \quad \mathbf{Q}(\mathbf{C} \mid \mathbf{A}; \mathbf{P}) := \sum_{i,j=1}^N (\mathbf{A}_{ij} - \mathbf{P}_{ij}) \delta(\mathbf{c}_i, \mathbf{c}_j) \quad [8]$$

113 where  $\mathbf{P}$  is the adjacency matrix of the null network and  
114  $\delta(\mathbf{c}_i, \mathbf{c}_j)$  is the Kronecker delta function. Then, we can state  
115 the modularity-maximization problem as follows:

$$116 \quad \max_{\mathbf{c} \in \mathbf{C}} \sum_{i,j=1}^N (\mathbf{A}_{ij} - \mathbf{P}_{ij}) \delta(\mathbf{c}_i, \mathbf{c}_j) \quad [9]$$

117 where modularity matrix  $\mathbf{B}$  can be defined as  $\mathbf{A} - \mathbf{P}$  (5).  
118

**D. Multilayer modularity maximization.** Consider an  $\mathbf{N}|\mathcal{T}|$ -node  
119 multilayer network, where  $\mathcal{T} = \{\mathbf{A}_1, \dots, \mathbf{A}_{|\mathcal{T}|}\}$  represents  
120 adjacency matrix for a layer  $\mathbf{A}_s$ . we can state the  
121 multilayer modularity-maximization problem

$$122 \quad \max_{\mathbf{c} \in \mathbf{C}} \sum_{i,j=1}^{N|\mathcal{T}|} \mathbf{B}_{ij} \delta(\mathbf{c}_i, \mathbf{c}_j) \quad [10]$$

123 where  $\mathbf{B}$  is the multilayer modularity matrix

$$124 \quad \mathbf{B} = \begin{bmatrix} B_1 & \omega \mathbf{I} & 0 & \dots & 0 \\ \omega \mathbf{I} & \ddots & \ddots & \ddots & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & \ddots & \omega \mathbf{I} \\ 0 & \dots & 0 & \omega \mathbf{I} & B_{|\mathcal{T}|} \end{bmatrix} \quad [11]$$

125 and  $\omega \in \mathbf{R}$  is the value of the inter-layer edge weight. We can  
126 rewrite the multilayer modularity-maximization problem in 10  
127 as:

$$128 \quad \max_{\mathbf{c} \in \mathbf{C}} \left[ \sum_{s=1}^{|\mathcal{T}|} \sum_{i,j=1}^{N|\mathcal{T}|} \mathbf{B}_{ijs} \delta(\mathbf{c}_{is}, \mathbf{c}_{js}) + 2\omega \sum_{s=1}^{|\mathcal{T}|-1} \sum_{i=1}^N \delta(\mathbf{c}_{is}, \mathbf{c}_{i,s+1}) \right] \quad [12]$$

129 where  $\mathbf{B}_{ijs}$  denotes the  $(i,j)$ th entry of  $\mathbf{B}_s$  (5).  
130

**E. Supracentrality Matrix Construction.** A supracentrality  
131 matrix  $\mathbf{C}(\omega) \in \mathbb{R}^{NT \times NT}$  is the matrix from which eigenvectors  
132 will be calculated, where  $\mathbf{N}$  represents the number of nodes  
133 in the network, and  $\mathbf{T}$  denotes the number of time layers for  
134 temporal networks. Given a sequence of adjacency matrices  
135  $\mathbf{A}^{(t)} \in \mathbb{R}^{N \times N}$  for  $t \in \{1, 2, \dots, T\}$ , we will turn them into  
136 corresponding centrality matrices  $\mathbf{C}$  (the function is related  
137 to the desired centrality measure, for example, eigenvector  
138 centrality uses  $\mathbf{C}(\mathbf{A}) = \mathbf{A}$ ). Then, these centrality matrices

139  $\mathbf{C}^{(t)}$  are coupled together. We define an interlayer-adjacency  
 140 matrix  $\tilde{\mathbf{A}} \in \mathbb{R}^{T \times T}$ , where each entry  $\tilde{A}_{tt'}$  encodes the inter-  
 141 layer coupling from layer  $t$  to layer  $t'$ ,  $\tilde{\mathbf{A}}$  can be either directed or undirected. The parameter  $\omega \geq 0$  then controls the strength  
 142 of connections between the layers. We will formally define the  
 143 supracentrality matrix  
 144

$$\mathbf{C}(\omega) = \begin{bmatrix} \mathbf{C}_1 & \mathbf{0} & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{C}_2 & \mathbf{0} & \dots \\ \mathbf{0} & \mathbf{0} & \mathbf{C}_3 & \dots \\ \vdots & \vdots & \ddots & \ddots \end{bmatrix} + \omega \begin{bmatrix} \tilde{\mathbf{A}}_{11}\mathbf{I} & \tilde{\mathbf{A}}_{12}\mathbf{I} & \tilde{\mathbf{A}}_{13}\mathbf{I} & \dots \\ \tilde{\mathbf{A}}_{21}\mathbf{I} & \tilde{\mathbf{A}}_{22}\mathbf{I} & \tilde{\mathbf{A}}_{23}\mathbf{I} & \dots \\ \tilde{\mathbf{A}}_{31}\mathbf{I} & \tilde{\mathbf{A}}_{32}\mathbf{I} & \tilde{\mathbf{A}}_{33}\mathbf{I} & \dots \\ \vdots & \vdots & \vdots & \vdots \end{bmatrix}, \quad [13]$$

145 where the second part is the Kronecker product of  $\tilde{\mathbf{A}}$  and  $\mathbf{I}$   
 146 (6).  
 147

148 **F. Joint and Conditional Centralities.** Let  $\mathbf{v}(\omega)$  be the calculated right dominant eigenvector that represents the centrality scores. The joint centrality of a node  $i$  in time layer  $t$ , denoted  $W_{it}(\omega)$ , is defined as

$$152 \quad W_{it}(\omega) = \mathbf{v}_{N(t-1)+i}(\omega), \quad [14]$$

153 this joint centrality score emphasizes the importance of both  
 154 the node  $i$  and the time layer  $t$ .

155 Given the joint centralities  $\{W_{it}(\omega)\}$ , we can thus define  
 156 the conditional centralities of nodes as follows

$$157 \quad Z_{it} = W_{it}(\omega) / \sum_i W_{it}(\omega), \quad [15]$$

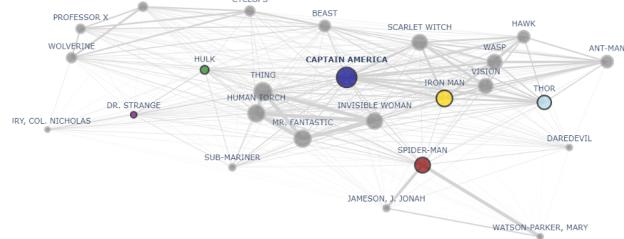
158 this conditional centrality of nodes is conditioned on layer  
 159  $t$ , so it can reflect the relative importance of nodes within a  
 160 layer (6).

## 161 Methods and Models

162 **Data Pre-processing.** The raw data we obtained contains all  
 163 Marvel comics before 2002 (7). This dataset works for our  
 164 initial attempts to find the most popular heroes, but we need a  
 165 smaller subset of it for our detailed analysis, due to restriction  
 166 in time and our resources.

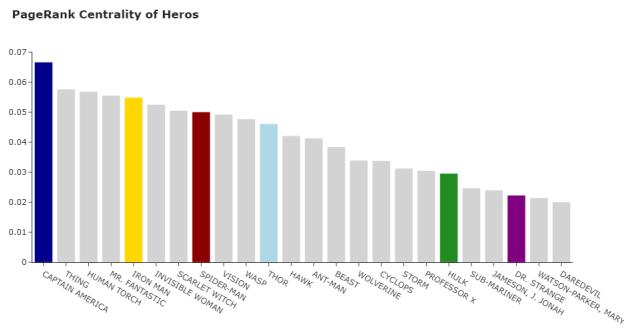
167 We decided to analyze only the Avengers series, which  
 168 includes volumes 1, 2, and 3, and the annuals of the years  
 169 before 2002. In order to do so, we use the bipartite graph  
 170 between heroes and comics and first filter out all comics that  
 171 we will not use. Then we check the publication dates of the  
 172 comics to divide them into the four eras - silver age, bronze  
 173 age, modern age, and heroes relaunched (8). We project each  
 174 network onto the hero-nodes to create a weighted hero-hero  
 175 network, in which the weight is the number of issues the heroes  
 176 occur together.

177 **Data Visualizations.** We begin our Exploratory Data Analysis  
 178 (EDA) process with a visualization of our network. In our first  
 179 visualization of the general dataset, each node represents a  
 180 hero, and each edge represents the interaction between two  
 181 heroes when they appear in the same issue. This is a weighted  
 182 undirected network, with the weight of the edge representing  
 183 the number of common issues. In figure 1, we visualize the  
 184 network dataset with only the top 25 nodes with the highest  
 185 degree.



186 **Fig. 1.** The visualization of top 25 heroes network. Each node represents a hero, and  
 187 each edge represents the interaction between two heroes when they appear in the  
 188 same issue. The size of the node represents the degree centrality  
 189

186 We further visualized PageRank, eigenvector, degree, and  
 187 closeness centralities as a part of our exploratory process.  
 188 Figure 2 is an example of our visualizations for the centrality  
 189 measures.



190 **Fig. 2.** Top 20 heroes PageRank Centrality. We see that Captain America has the  
 191 highest PageRank Centrality follow by the Thing  
 192

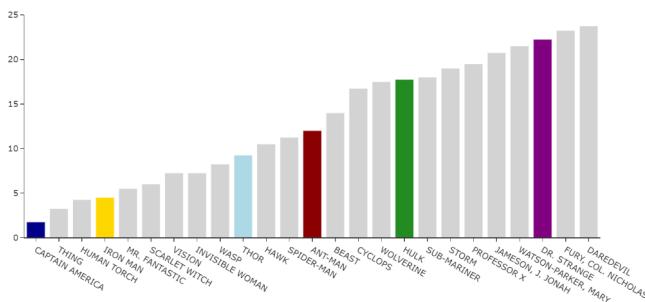
193 However, because each centrality measure represents different  
 194 information, we need to find a way to show a summary of  
 195 the 4 centralities we used. As a result, we decided to use a  
 196 measure called average centrality rank. The average centrality  
 197 rank is equal to the mean of the ranks of PageRank, eigenvector  
 198 centrality, degree, and closeness centralities. For example, Captain  
 199 America is ranked 1st in PageRank, ranked 4th in Eigenvector  
 200 centrality, ranked 1st in degree centrality, and ranked 1st in  
 closeness centrality. So Captain America's average centrality  
 rank is 1.75. The measures is also demonstrated in table 1  
 below.

201 **Table 1. Demonstration of Centrality Measures we used**

	Captain America	Thing
PageRank Centrality	0.066699	0.057688
Eigenvector Centrality	0.312066	0.318561
Degree Centrality	4409.0	3828.0
Closeness Centrality	141.338880	104.086414
PageRank Ranking	1	2
Eigenvector Ranking	4	1
Degree Ranking	1	8
Closeness Ranking	1	2
Average Centrality Rank	1.75	3.25

We compute the 20 lowest average centrality rank to obtain

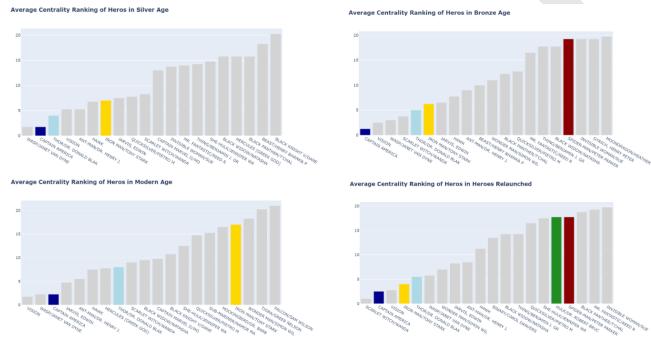
202 the list of heroes with the highest centralities overall, as shown  
203 in figure 3 below.



**Fig. 3.** Top 20 Heroes with Lowest Average Centrality Rank. The lowest is Captain America followed by the Thing, this means that Captain America has the Highest centrality overall, followed by the Thing.

204 We double check on the heroes with top 10 centrality ranking  
205 and find out that all of them exist in all four layers, so we  
206 decide to further filter our network to include only the nodes  
207 that occur on all four layers. This can make later calculations  
208 that require all nodes to exist on all layers possible, and we  
209 will not end up with a lot of zero entries if we were to add all  
210 nodes in all layers to each layer.

211 **Temporal Network Centrality Visualizations.** Once we obtained  
212 our temporal network, we decided to apply the same  
213 algorithm to compute their Average Centrality Ranking, so  
214 that we could get an overall idea of how heroes' centrality  
215 changes over time. Figure 4 shows four visualizations of  
216 Average Centrality Ranking as we move from Silver Age to Heroes  
217 Age.



**Fig. 4.** Average Centrality Rank of each layer of the Temporal Network. We see that as we move from layers to layers, the average centrality ranking changes

218 **Community detection in multilayer networks.** We apply  
219 community detection by maximizing the modularity function in  
220 multilayer networks to our data with `Genlouvain` package for  
221 Matlab (9). To simplify our study, we choose the coupling  
222 parameter  $\omega$  as 0.5.

223 **Supracentrality Analysis with Temporal Network.** Previously,  
224 we computed basic centrality measures over our datasets, and  
225 we could see some changes in the rankings, but it lacked a  
226 time component. In an attempt to dig deeper into the effects  
227 of time on character importance, we decided to proceed to

228 perform a supracentrality analysis on this temporal character  
229 network.

230 After constructing the supracentrality matrix  $\mathbb{C}(\omega)$ , we then  
231 compute the dominant eigenvector  $\mathbf{v}(\omega)$ , and use its entries  
232 as scores for centrality measures. Thus, we solve the following  
233 equation

$$\mathbb{C}(\omega)\mathbf{v}(\omega) = \lambda_{max}(\omega)\mathbf{v}(\omega), \quad [16]$$

234 and  $\mathbf{v}(\omega)$  would contain all the centrality scores (6, 10, 11). 235

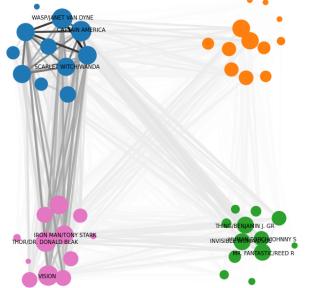
## Results

236 We present the results of our numerical experiments with  
237 different methods, respectively. 238

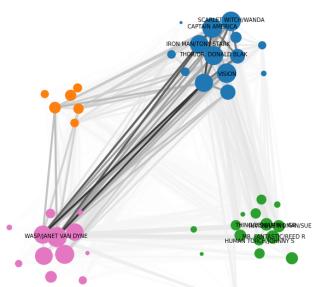
239 **Community detection result visualization.** Figure 5a and Figure 240  
241 5b depict a partition into 4 communities that we found with  
242  $\omega = 0.5$ , while Figure 5c and Figure 5d depict a partition  
243 into 3 communities. We also present the resulting communities  
244 of the top 10 heroes in Figure 6. 245

245 **Community detection result interpretation.** Here are the  
246 results from community detection 9. For each layer that  
247 represents a period, all forty-seven heroes are sorted into four or  
248 three communities. We now analyze the community formation  
249 of all four ages and the top ten centrality heroes. In the Silver  
250 era, which is the earliest ear in our dataset, we found four  
251 communities with relatively obvious traits. For instance, the  
252 heroes in group one all have learned martial arts. Captain  
253 America taught most of them, such as Wanda. Most of the  
254 heroes in group two have different states, and most heroes  
255 in group three have objects as their weapons, such as Thor's  
256 hammer and Ironman's armor. The heroes in the last group  
257 all experienced mutation, such as the Hulk, infected by gamma  
258 rays. What's more, looking at the heroes with the top ten  
259 centrality ranking, we notice that they are evenly distributed  
260 in three different communities. To sum up, at the very earliest  
261 age of the marvel comic era, which can be seen in figure 5a,  
262 most of the heroes appear in the same scene because they  
263 share commonalities in background or ability. However, such  
264 a trend started to change when we look at the following eras.  
265 In the bronze age, there is hardly any obvious traits for each  
266 community 10. The top ten heroes are mostly gathered into  
267 two communities as 5b shows. The connections between the  
268 most popular characters have a clear increase. Such a trend  
269 continues in the next two eras. In the Modern age 11, the  
270 number of communities has decreased from four to three 5c,  
271 and in the Heroes age 12, all ten top heroes have gathered into  
272 two out of the three communities 5d. Overall, as time goes by,  
273 marvel tends to create more interactions between the more  
274 popular heroes instead of following the more logical storyline  
275 base on the background, which is a widely used strategy for  
276 commercial works. 277

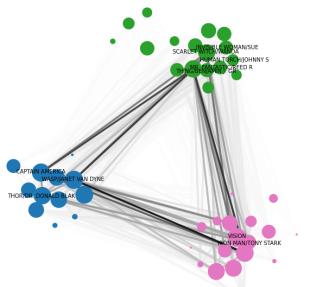
277 **Supracentrality Analysis with Temporal Network.** To better  
278 understand changes in the top 10 characters' importance over  
279 time, we performed PageRank Supracentrality with teleportation  
280 parameter  $\sigma = 0.85$ , and  $\omega \in \{1, 10, 100, 1000\}$  in Figure  
281 7a and Figure 7b. We then obtained the results from performing  
282 Eigenvector Supracentrality with  $\omega \in \{1, 10, 100, 1000\}$  in Figure  
283 8a and Figure 8a. All experiments were done with undirected  
284 interlayer coupling because we noticed that directed  
285



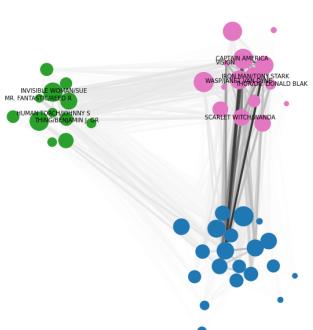
(a) Silver Age



(b) Bronze Age

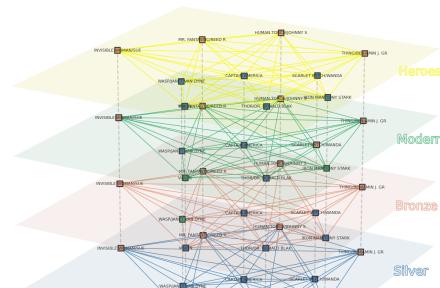


(c) Modern Age



(d) Heroes Age

**Fig. 5.** The visualization of communities we found in four eras. The node color represents different communities and the node size is proportional to its centrality.



**Fig. 6.** The visualization of communities we found in four eras in a 3D view of all layers. The node color represents different communities (same color in different layers do not share similarities, color is only used to differentiate nodes in the same layer). Intralayer edges have different colors for different layers; this is not related to color of nodes either, but only used to differentiate different layers.

interlayer coupling gives the largest importance to the earlier time periods, and it is not easy to observe the temporal changes given we only have four time periods.

We vary the strength of coupling parameter  $\omega$  to see how it affects the temporal changes in centrality measures. One interesting thing to note is when  $\omega$  becomes very large, the conditional centralities become very smooth, almost straight lines as shown in Figure 7b when  $\omega = 1000$ .

Comparing the supracentrality analysis results with the basic centrality results, we found that Captain America remains the most important hero throughout the Avengers period. Although some heroes are more important in the Avengers Bronze period like Vision, they are not so prominent in the overall basic centrality analysis performed previously.

In both PageRank and Eigenvector centrality analysis, a trend we noticed is that there seems to be a large rise in hero importance during the Bronze Period. This could be explained by the large increase in the volumes of comics during the Bronze Period, leading to more character co-appearances, and potentially more complex character interactions.

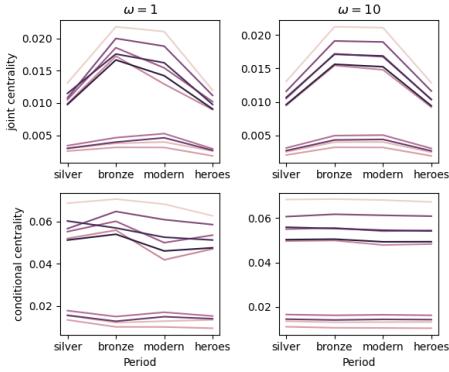
## Limitations

First, to simplify our study, when we construct the multilayer network for further study, we choose to consider characters who appear in all four ages and thus it is likely to lose important information due to this decision.

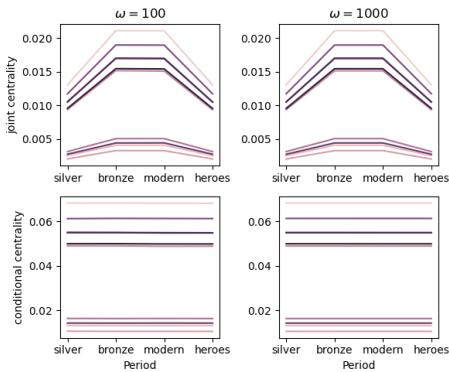
For our approach of community detection in multilayer network, we choose the inter-layer coupling  $\omega$  to be 0.5 without rigorous numerical experiment. Normally, the highest achievable value of persistence for an optimal partition obtained with a given value of  $\omega$  is a non-decreasing function in  $\omega$  (5). The choice of inter-layer coupling might affect the reasonability of partitions based on our data set.

## Conclusions and Discussion

As we enter the Modern Age, we note a significant change in community structure: the four communities in previous eras condense into three communities, and interactions between



(a) PageRank Result with small  $\omega$

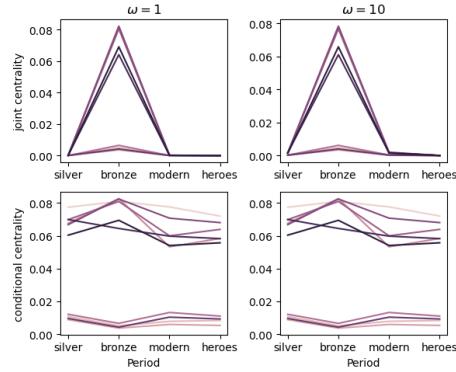


(b) PageRank Result with large  $\omega$

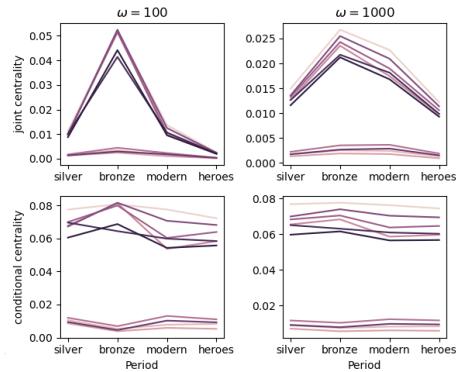
CAPTAIN AMERICA  
 THING/BENJAMIN J. GR  
 HUMAN TORCH/JOHNNY S  
 IRON MAN/TONY STARK  
 MR. FANTASTIC/CREED R  
 SCARLET WITCH/WANDA  
 VISION  
 INVISIBLE WOMAN/SUE  
 WASP/PLANET VAN DYNE  
 THOR/DR. DONALD BLAK

(c) Legend

**Fig. 7.** Results of performing supracentrality analysis using PageRank centrality, with teleportation parameter  $\sigma = 0.85$ , and varying the strength of coupling parameter  $\omega \in \{1, 10, 100, 1000\}$ .



(a) Eigenvector Centrality Result with small  $\omega$



(b) Eigenvector Centrality Result with large  $\omega$

**Fig. 8.** Results of performing supracentrality analysis using Eigenvector centrality varying the strength of coupling parameter  $\omega \in \{1, 10, 100, 1000\}$ .

Therefore, despite being time-consuming, these studies were able to produce more accurate predictions.

## Code

Readers can find our code from our [Github repository](#).

## Acknowledgement

We want to thank our Professor Mason Porter for the helpful advice he gives regarding our topic and method choice, and the papers that he recommends to us. We also thank the authors of our data sets.

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320 communities are more frequent. One possible explanation for  
 321 this is that comics entered a “dark age” (8) in which a shift  
 322 in fan preferences steered productions to focus on dark or  
 323 anti-heroes such as Scarlet Witch or Wolverine. This stylistic  
 324 shift also added vigilante elements to traditional characters.  
 325 As a result of a more similar character design of this era, com-  
 326 munity structure changed accordingly.

327 One limitation of the Louvain algorithm is how it searches in  
 328 the space of solutions. Although the modularity function is  
 329 deterministic, a consistent solution is not guaranteed since the  
 330 algorithm can completely miss it. This may have contributed  
 331 to some of the random noises in our results which deviate from  
 332 real-world Marvel hero relationships.

333 Contrary to our anticipation, some characters whom we be-  
 334 lieved to take minor roles in fact have higher centralities than  
 335 some major characters. Due to limitations in time and re-  
 336 sources, we weren’t able to label weights according to character  
 337 relationships in a careful manner. Ideally, characters who are  
 338 for example, siblings, should be weighted higher than char-  
 339 acters who are just friends. However, this involves carefully  
 340 reading the comics and observing character interaction, which  
 341 were carried out by many sophisticated studies we mentioned.

## Appendix

Layer 1		Silver	
ANT-MAN/DR. HENRY J.	ATTUMA	BEAST/HENRY &HANK& P	COUNT NEFARIA, LUCHI
BLACK WIDOW/NATASHA	CAPTAIN MARVEL II/MO	BINARY/CAROL DANVERS	CRYSTAL [INHUMAN]
CAPTAIN AMERICA	DR. DRUID/ANTHONY LU	BLACK KNIGHT V/DANE	DAREDEVIL/MATT MURDO
GRIM REAPER/ERIC WIL	FALCON/SAM WILSON	BLACK PANTHER/T'CHAL	HULK/DR. ROBERT BRUC
HAWK	HELLCAT/PATSY WALKER	GYRICH, HENRY PETER	HUMAN TORCH/JOHNNY S
HERCULES [GREEK GOD]	MOCKINGBIRD/DR. BARB	IRON MAN/TONY STARK	ICEMAN/ROBERT BOBBY
POWER MAN/ERIK JOSTE	MOONDRAKON/HEATHER D	JARVIS, EDWIN	INVISIBLE WOMAN/SUE
QUICKSILVER/PIETRO M	O'BRIEN, MICHAEL	PHARAOH RAMA-TUT	LUNA/LUNA MAXIMOFF
SCARLET WITCH/WANDA	SHE-HULK/JENNIFER WA	SPIDER-MAN/PETER PARKER	MR. FANTASTIC/REED R
SWORDSMAN/JACQUES DU	SUB-MARINER/NAMOR MA	STARFOX/EROS	THING/BENJAMIN J. GR
WASP/JANET VAN DYNE	TIGRA/GREER NELSON	THING/BENJAMIN J. GR	
	WIZARD/BENTLEY WITTM	ULTRON	
		VISION	
		WONDER MAN/SIMON WIL	
<b>martial arts</b>	<b>different state</b>	<b>has special object</b>	<b>mutations</b>

**Fig. 9.** This chart includes the four communities of the Silver era for the 47 heroes that this paper mainly focuses on. Each column represents a community. The order does not have any specific meanings. Heroes in four communities has some similar traits. Most of the heroes in group two have different states, and most heroes in group three have objects as their weapons. The heroes in the last group all have experienced mutation. The heroes that have the top ten average centrality are marked in green. They are evenly spread in three groups.

Layer 4		Heroes	
ATTUMA	ANT-MAN/DR. HENRY J.	DAREDEVIL/MATT MURDO	
BEAST/HENRY &HANK& P	BINARY/CAROL DANVERS	DR. DRUID/ANTHONY LU	
BLACK KNIGHT V/DANE	CAPTAIN AMERICA	GRIM REAPER/ERIC WIL	
BLACK PANTHER/T'CHAL	COUNT NEFARIA, LUCHI	HELLCAT/PATSY WALKER	
BLACK WIDOW/NATASHA	GYRICH, HENRY PETER	HULK/DR. ROBERT BRUC	
CAPTAIN MARVEL II/MO	IRON MAN/TONY STARK	HUMAN TORCH/JOHNNY S	
CRYSTAL [INHUMAN]	JARVIS, EDWIN	INVISIBLE WOMAN/SUE	
FALCON/SAM WILSON	PHARAOH RAMA-TUT	MOCKINGBIRD/DR. BARB	
HAWK	POWER MAN/ERIK JOSTE	MR. FANTASTIC/REED R	
HERCULES [GREEK GOD]	SCARLET WITCH/WANDA	SPIDER-MAN/PETER PARKER	
ICEMAN/ROBERT BOBBY	THOR/DR. DONALD BLAK	SWORDSMAN/JACQUES DU	
LUNA/LUNA MAXIMOFF [	ULTRON	THING/BENJAMIN J. GR	
MOONDRAKON/HEATHER D	VISION		
O'BRIEN, MICHAEL	WASP/JANET VAN DYNE		
QUICKSILVER/PIETRO M	WONDER MAN/SIMON WIL		
SHE-HULK/JENNIFER WA	STARFOX/EROS		
SUB-MARINER/NAMOR MA			
TIGRA/GREER NELSON			
WIZARD/BENTLEY WITTM			

**Fig. 12.** This chart includes the three communities of the Hero era for the 47 heroes that this paper mainly focuses on. Each column represents a community. No obvious commonality was spotted among any of the groups. The order does not have any specific meanings. The heroes that have the top ten average centrality are marked in green. They all gathered in two of the communities.

Layer 2		Bronze	
BLACK KNIGHT V/DANE	BLACK WIDOW/NATASHA	ANT-MAN/DR. HENRY J.	CAPTAIN MARVEL II/MO
BLACK PANTHER/T'CHAL	HELLCAT/PATSY WALKER	ATTUMA	CRYSTAL [INHUMAN]
CAPTAIN AMERICA	HERCULES [GREEK GOD]	BEAST/HENRY &HANK& P	HUMAN TORCH/JOHNNY S
DR. DRUID/ANTHONY LU	ICEMAN/ROBERT BOBBY	BINARY/CAROL DANVERS	INVISIBLE WOMAN/SUE
GRIM REAPER/ERIC WIL	MOONDRAKON/HEATHER D	DAREDEVIL/MATT MURDO	LUNA/LUNA MAXIMOFF [
HAWK	PHARAOH RAMA-TUT	FALCON/SAM WILSON	MOCKINGBIRD/DR. BARB
HULK/DR. ROBERT BRUC	GYRICH, HENRY PETER	MR. FANTASTIC/REED R	
IRON MAN/TONY STARK	JARVIS, EDWIN	O'BRIEN, MICHAEL	
MOCKINGBIRD/DR. BARB	SUB-MARINER/NAMOR MA	SHE-HULK/JENNIFER WA	
SCARLET WITCH/WANDA	POWER MAN/ERIK JOSTE	SPIDER-MAN/PETER PARKER	
STARFOX/EROS	WONDER MAN/SIMON WIL	WIZARD/BENTLEY WITTM	
QUICKSILVER/PIETRO M	ULTRON	TIGRA/GREER NELSON	
THOR/DR. DONALD BLAK	VISION	THING/BENJAMIN J. GR	
WASP/JANET VAN DYNE	SWORDSMAN/JACQUES DU		

**Fig. 10.** This chart includes the four communities of the Bronze era for the 47 heroes that this paper mainly focuses on. Each column represents a community. No obvious commonality was spotted among any of the groups. The order does not have any specific meanings. The heroes that have the top ten average centrality are marked in green. Nine out of ten of them are gathered in two of the groups.

Layer 3		Modern	
ATTUMA	ANT-MAN/DR. HENRY J.	DAREDEVIL/MATT MURDO	
BLACK KNIGHT V/DANE	BEAST/HENRY &HANK& P	FALCON/SAM WILSON	
CAPTAIN AMERICA	BINARY/CAROL DANVERS	HAWK	
CAPTAIN MARVEL II/MO	BLACK PANTHER/T'CHAL	HELLCAT/PATSY WALKER	
DR. DRUID/ANTHONY LU	BLACK WIDOW/NATASHA	HULK/DR. ROBERT BRUC	
JARVIS, EDWIN	COUNT NEFARIA, LUCHI	HUMAN TORCH/JOHNNY S	
POWER MAN/ERIK JOSTE	CRYSTAL [INHUMAN]	INVISIBLE WOMAN/SUE	
SHE-HULK/JENNIFER WA	GRIM REAPER/ERIC WIL	MOCKINGBIRD/DR. BARB	
STARFOX/EROS	GYRICH, HENRY PETER	MOONDRAKON/HEATHER D	
SUB-MARINER/NAMOR MA	HERCULES [GREEK GOD]	MR. FANTASTIC/REED R	
THOR/DR. DONALD BLAK	ICEMAN/ROBERT BOBBY	O'BRIEN, MICHAEL	
WASP/JANET VAN DYNE	IRON MAN/TONY STARK	SCARLET WITCH/WANDA	
	LUNA/LUNA MAXIMOFF [	SPIDER-MAN/PETER PARKER	
	PHARAOH RAMA-TUT	THING/BENJAMIN J. GR	
	QUICKSILVER/PIETRO M	TIGRA/GREER NELSON	
	SWORDSMAN/JACQUES DU	WIZARD/BENTLEY WITTM	
	ULTRON	WONDER MAN/SIMON WIL	
	VISION		

**Fig. 11.** This chart includes the three communities of the Modern era for the 47 heroes that this paper mainly focuses on. Each column represents a community. No obvious commonality was spotted among any of the groups. The order does not have any specific meanings. The heroes that have the top ten average centrality are marked in green. They evenly spread among three communities.